would be that of a square plate made up of the two end sections of the rectangle. The second sub-problem would be that of a strip of infinite length (i.e. without ends).

The flow field characteristics for a circular plate are somewhat different from those of planforms having corners. In particular, as illustrated in Fig. 4, partition lines are absent. While the flow paths are perpendicular to the edges of the plate, they are convergent rather than parallel. The convergence of the paths tends to accelerate the flow as it moves inward toward the center of the plate. The central region of the plate is dominated by a billowing plume.

In the view of the authors, the flow field information suggests that generalization of existing (albeit conflicting) square plate heat-transfer correlations to other planforms may involve more than simply using the short side as the characteristic dimension [4].

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BOUNDARY-LAYER ANALYSIS OF THE FLOW BOILING CRISIS

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NOMENCLATURE

B, $= G^+/(f/2)$;
C, constant;
D_e, equivalent diameter [ft];
f, friction factor;
G, mass velocity [lb/h ft^2];
H_{fg}, heat of evaporation [Btu/lb];
p, pressure [lb/in^2];
q'', heat flux [Btu/h ft^2];
Re, Reynolds number;
ΔT_{sc}, subcooling [°F];
U_o, main stream velocity [ft/s];
u, flow velocity parallel to wall [ft/s];
V, flow velocity normal to wall [ft/s];
y, distance normal to wall [ft];
x, distance parallel to wall [ft].

Greek symbols

α, void fraction;
δ, thickness of boundary layer [ft];
ρ, density [lb/ft^3];
τ, shear [lb/ft^2];
μ, viscosity [lb/h ft];
χ, steam quality.

Subscripts

δ, refers boundary-layer thickness;
j, refers injection flow;
l, refers saturated liquid condition;
L, refers laminar flow;
M, refers mixture;
o, refers main stream properties;
T, refers turbulent;
TP, refers two-phase flow;
v, refers saturated vapor condition;
w, refers wall condition;
lam, refers laminar sublayer condition;
sat, refers saturated condition;
crit, refers boundary-layer separation condition;
inj, refers permeable flat plate;
non-inj, refers non-permeable flat plate.

Superscripts

+, refers non-dimensional quantity.

1. INTRODUCTION

THE FLOW boiling crisis can be considered primarily as a hydrodynamic phenomenon. The process begins with flow stagnation under the boundary layer when the layer separates from the heated wall. Due to the high heat flux at the surface, the stagnant liquid evaporates resulting in a vapor blanket on the heated wall. When the local wall temperature exceeds the Leidenfrost point, the flow boiling crisis occurs.

Kutateladze [1–3] has suggested an analysis of the flow boiling crisis using the concept of boundary-layer separation (blow-off or break-away) from a permeable flat plate with gas injection. In order to correlate the experimental flow boiling crisis data, Kutateladze [1] had to add the boundary-layer separation heat flux to the pool boiling critical heat flux (computed from Helmholtz instability) to form the flow boiling critical heat flux as:

$$q''_{\text{crit flow boiling}} = q''_{\text{B.L. separation}} + q''_{\text{crit pool boiling}}.$$

The concepts of flow boundary layer and pool boiling are obviously inconsistent and there is no reason to believe these heat fluxes can be additive. The data correlation thus obtained cannot be regarded as an adequate demonstration of the validity of this new boundary-layer analysis. The purpose of this paper is to correlate the critical heat flux data by using only the concept of boundary-layer separation.

2. ANALYSIS

Kutateladze [1] gives the critical condition of boundary-layer separation from a flat plate with isothermal injection of the same fluid (see equation A-7 of the Appendix) as

$$(\rho_j V_j)_{\text{crit}} = 2f\rho_o U_o \tag{1}$$

where f is the friction factor for no injection, subscript j refers to the injected fluid, subscript o refers to the main stream fluid, V is the velocity normal to main stream and U is the velocity parallel to main stream.

The above equation has been verified by the data of Hacker [4] who reported that the boundary layer separates at

$$\rho_j V_j = 0{\cdot}01\ \rho_o U_o.$$

In subcooled boiling, the separation of the boundary layer may cause a boiling crisis. This phenomenon has been observed by the author in a subcooled Freon boiling as shown in Fig. 1.

When the critical injection momentum equals to the momentum of the vapor generated by boiling, equation (1) is rewritten by the present author as:

$$\frac{1}{\rho_v}\left(\frac{q''_{\text{crit}}}{H_{fg}}\right)^2 = \rho_j V_j^2 = \rho_l U_o^2 (2f)^2 \left(\frac{\rho_l}{\rho_j}\right) \tag{2}$$

where

$$\rho_l = \rho_o = \rho_j.$$

Rearranging and substituting ρ_v for ρ_j during surface boiling, equation (2) becomes

$$q''_{\text{crit}} = 2fH_{fg}\rho_l U_o. \tag{3}$$

The flow friction of a channel having subcooled local boiling behaves similarly to that of a channel with a rough surface. The velocity profile for turbulent flow in a rough channel which has been reported by Tyul'panor [5], lies between the velocity profiles of turbulent and laminar flow. Since the friction factor of fully developed turbulent flow in a smooth tube is

$$f_T \propto Re^{-0{\cdot}2} \tag{4}$$

and the friction factor of a fully developed laminar flow in a smooth tube is

$$f_L \propto Re^{-1{\cdot}0} \tag{5}$$

it is reasonable to assume the friction factor of a turbulent flow in a rough channel or of a two-phase flow in local boiling to be

$$f_{TP} \propto Re^{-0{\cdot}6} \tag{6}$$

where

$$Re = (\rho_l U_o D_e/\mu_{\text{sat}}).$$

The above equation has been verified by limited amount of air–water data [6] at low void fractions.

When equation (6) is substituted into equation (3), it becomes

$$q''_{\text{crit}} = CH_{fg}\rho_l U_o/Re^{0{\cdot}6}. \tag{7}$$

Comparison of the above equation with the existing flow boiling crisis data in water at 1000–2000 lb/in^2 inside a single tube test section with uniform heat flux, shows C to be a function of the bulk quality or the degree of subcooling. Equation (7) then becomes

$$q''_{\text{crit}} = 1{\cdot}76 - 7{\cdot}433\chi + 12{\cdot}222\chi^2 \frac{H_{fg}\rho_l^{0{\cdot}4}}{D_e^{0{\cdot}6}} U_o^{0{\cdot}4}\mu_{\text{sat}}^{0{\cdot}6} \tag{8}$$

where χ is the bulk quality, χ is negative in a subcooled local boiling.

The comparison of the predictions of equation (8) with uniform flux–water boiling crisis data, which are the same data used in [7], is given in Fig. 2. The predictions of equation (8) are also compared with the subcooled, uniform flux data from Orthoterphenyl [8] as shown in Fig. 3.

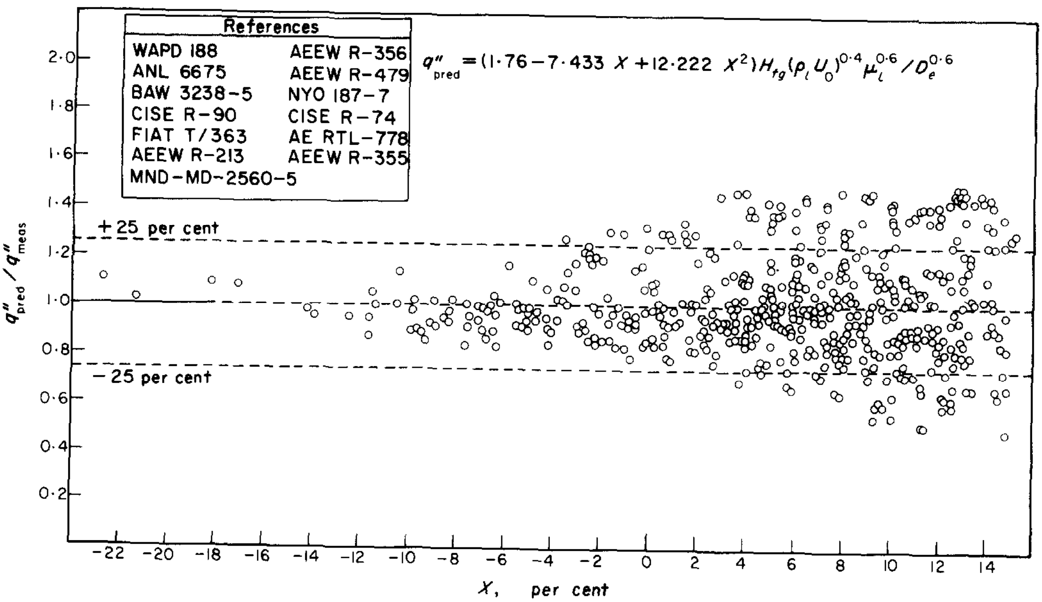

FIG. 2. Comparison of predictions with uniform flux boiling crisis data from water.

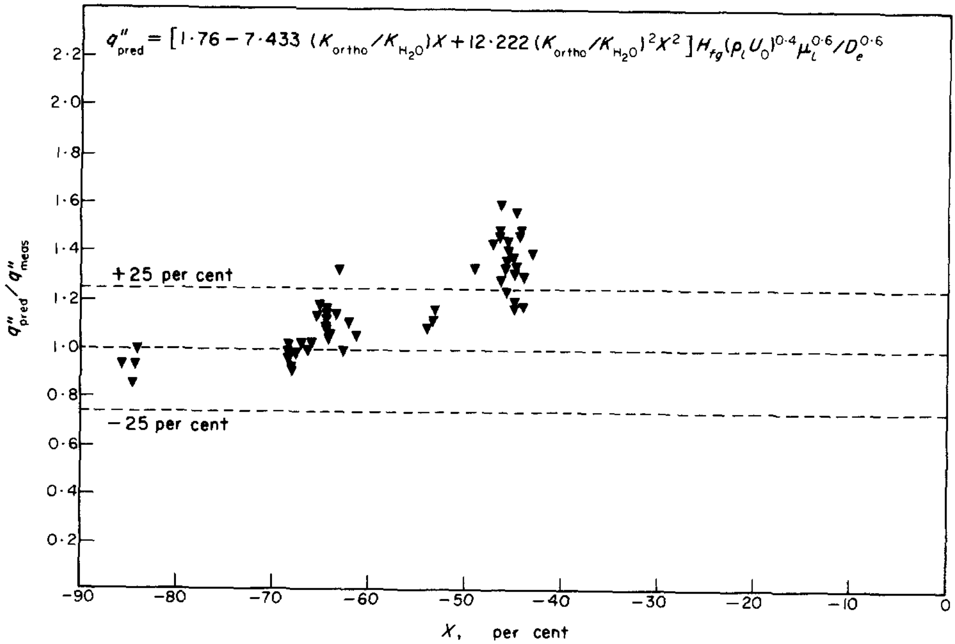

FIG. 3. Comparison of predictions with subcooled boiling crisis data from Orthoterphenyl.

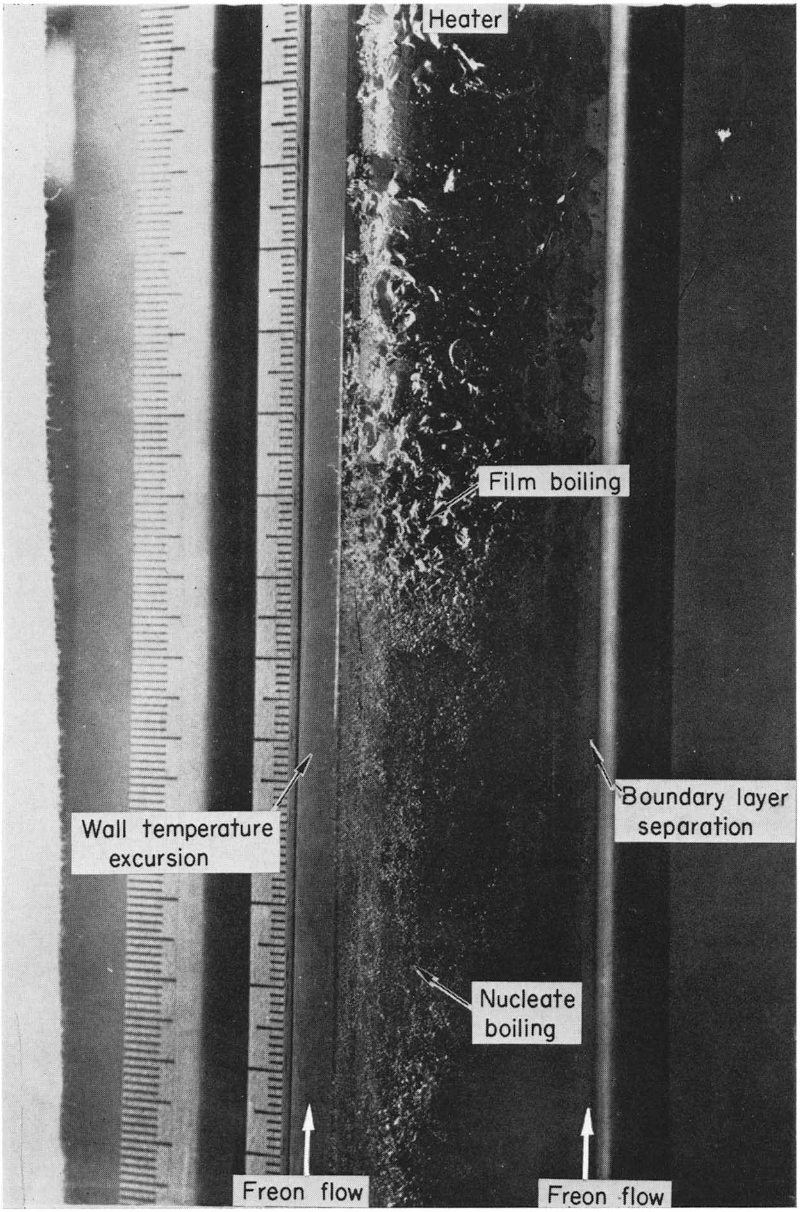

FIG. 1. Boiling crisis in subcooled freon flow. $p = 40$ lb/in^2; $G = 1{\cdot}5 \times 10^6$ lb/h ft^2; $\Delta T_{sc} = 30°$ F at boiling crisis.

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APPENDIX

Boiling Flow Boundary-layer Separation

The effect of bubble generation from a boiling surface on the boundary-layer resembles that of vapor injection through a permeable wall. Kutateladze [3] has developed equations for the turbulent bubble boundary layer over a flat permeable wall.

Those equations are rearranged for clarity and presented in the following: For high Reynolds numbers the laminar sublayer is so thin as to be negligible. The shear stress in a flow with injection is given [3] as:

$$\tau^+_{\text{inj}} = \tau^+_{\text{non-inj}}(1 + Bu^+) \tag{A-1}$$

where

$$B = \frac{G^+}{f/2}$$

$$\tau^+_{\text{inj}} = \tau_{\text{inj}}/\tau_{\text{w, inj}}$$

$$G^+ = G_{\text{inj}}/G_o$$

$$u^+ = u/U_o.$$

Subscript o refers the potential flow outside the boundary-layer. In the turbulent boundary-layer, the mixing length concept applies.

$$\sqrt{\left(\frac{\tau}{\rho}\right)} = 0{\cdot}4y\frac{\partial u}{\partial y}.$$

This may be rewritten as

$$\sqrt{\left(\frac{\tau^+\tau_w}{\rho^+\rho_o U_o^2}\right)} = 0{\cdot}4y^+\frac{\partial u^+}{\partial y^+} \tag{A-2}$$

where

$$\rho^+ = \rho/\rho_0$$

$$y^+ = y/\delta.$$

Substituting equation (A-1) into equation (A-2), we get

$$\sqrt{\left(\frac{\tau_w\tau^+_{\text{non-inj}}(1 + Bu^+)}{\rho^+\tau/(f_{\text{non-inj}}/2)}\right)} = 0{\cdot}4y^+\frac{\partial u^+}{\partial y^+}.$$

Rearrange,

$$\sqrt{\frac{\rho_o}{\rho}}\left(\frac{f}{2} + \frac{f}{2}Bu^+\right) = 0{\cdot}4y^+\frac{\partial u^+}{\partial y^+}. \tag{A-3}$$

Now let $1 \ll Bu^+$ at the boundary-layer separation.

$$\int\limits_{u^+_\delta}^{u^+} \frac{\partial u^+}{\sqrt{\left(\frac{\rho_o}{\rho}u^+\right)}} = \left[\sqrt{\left(\frac{f}{2}B_{\text{crit}}\right)}\right]\int\limits_{y^+_{\text{lam}}}^{y^+_\delta}\frac{\partial y^+}{0{\cdot}4y^+} \tag{A-4}$$

$$\int\limits_0^{u^+} \frac{\partial u^+}{\sqrt{\left(\frac{\rho_o}{\rho}u^+\right)}} = \frac{\sqrt{\left(\frac{f}{2}B_{\text{crit}}\right)}}{0{\cdot}4}(\ln y^+_\delta - \ln y^+_{\text{lam}}).$$

For $Re \to \infty, \ln y^+_\delta \to 0;\ -\ln y^+_{\text{lam}} \to \ln Re_\delta$

From experimental evidence, Kutateladze [3] reported that

$$\sqrt{\left(\frac{f_{\text{non-inj}}}{2}\right)} = \frac{0{\cdot}4}{\ln Re_\delta}.$$

Hence

$$\int\limits_0^{u^+} \frac{\partial u^+}{\sqrt{\left(\frac{\rho_o}{\rho}u^+\right)}} = \sqrt{B_{\text{crit}}}. \tag{A-5}$$

For a uniform injection flux at isothermal condition, $\rho = \rho_o$

$$\sqrt{(B_{\text{crit}})} = 2. \tag{A-6}$$

Since $B = 2G_{\text{inj}}/G_o\, f_{\text{non-inj}}$, equation (A-6) becomes

$$G_{\text{inj, crit}} = 2G_o f_{\text{non-inj}} \tag{A-7}$$

where

$G_{\text{inj}} = (\rho V)_{\text{inj}}$, V_{inj} is normal to wall

$G_o = \rho_o U_o$, U_o is velocity of main stream parallel to wall.